International Teenagers Mathematics Olympiad

10 – 14 December, 2015, Sungai Petani Kedah Malaysia

Individual Contest

Time limit: 120 minutes 2015/12/11

Instructions:

- Do not turn to the first page until you are told to do so.
- Remember to write down your team name, your name and contestant number in the spaces indicated on the first page.
- The Individual Contest is composed of two sections with a total of 120 points.
- Section A consists of 12 questions in which blanks are to be filled in and only **ARABIC NUMERAL** answers are required. For problems involving more than one answer, points are given only when **ALL** answers are correct. Each question is worth 5 points. There is no penalty for a wrong answer.
- Section B consists of 3 problems of a computational nature, and the solutions should include detailed explanations. Each problem is worth 20 points, and partial credit may be awarded.
- Diagrams are NOT drawn to scale. They are intended only as aids.
- You have a total of 120 minutes to complete the competition.
- No calculator, calculating device, watches or electronic devices are allowed.
- Answers must be in pencil or in blue or black ball point pen.
- All papers shall be collected at the end of this test.

*Name:*____________________ *No.*________ *Team:*____________

------- Jury use only -------

Section A Score												Total Score
1	2	3	4	5	6	7	8	9	10	11	12	
Section B Score												
1				2				3				

Individual Contest

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*Name:*__________________ *No.*__________ *Team:*______________ *Score:*______

Section A.

In this section, there are 12 questions. Fill in the correct answer on the space provided at the end of each question. Each correct answer is worth 5 points. Be sure to read carefully exactly what the question is asking.

1. Evaluate $M=\frac{1}{9-\sqrt{80}}-\frac{1}{\sqrt{80}-\sqrt{79}}+\frac{1}{\sqrt{79}-\sqrt{78}}-\cdots-\frac{1}{\sqrt{10}-3}$.

***Answer* :** ____________________

2. Find the smallest positive integer n such that both $2n$ and $3n+1$ are squares of integers.

***Answer* :** ____________________

3. How many different possible values of the integer a are there so that $||x-2|-|3-x||=2-a$ has solutions?

***Answer* :** ____________________

4. If $\sqrt{k-9}$ and $\sqrt{k+36}$ are both positive integers, what is the sum of all possible values of k?

***Answer* :** ____________________

5. Find the largest positive integer n such that the sum of the squares of the positive divisors of n is n^2+2n+2.

***Answer* :** ____________________

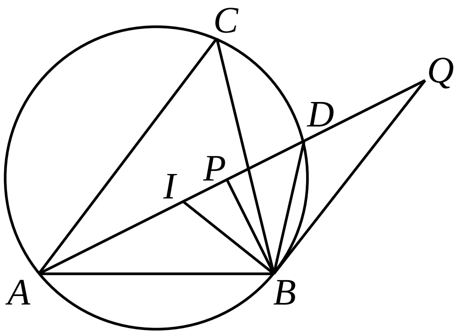
6. Find the smallest two-digit number such that its cube ends with the digits of the original number in reverse order.

***Answer* :** ____________________

7. A Mathematics test consists of 3 problems, each problem being graded independently with integer points from 0 to 10. Find the number of ways in which the total number of points for this test is exactly 21.

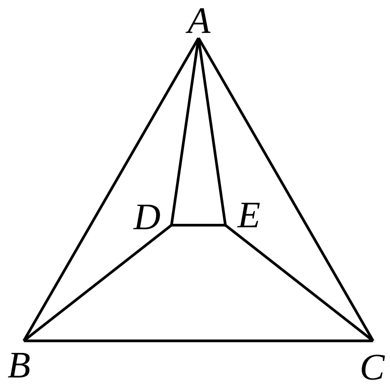
***Answer* :** ________________ ways

8. In the triangle ABC, the bisectors of $\angle CAB$ and $\angle ABC$ meet at the in-center I. The extension of AI meets the circumcircle of triangle ABC at D. Let P be the foot of the perpendicular from B onto AD, and Q a point on the extension of AD such that $ID = DQ$. Determine the value of $\dfrac{BQ \times IB}{BP \times ID}$.



Answer **:** ____________________

9. D and E are points inside an equilateral triangle ABC such that D is closer to AB than to AC. If $AD = DB = AE = EC = 7$ cm and $DE = 2$ cm, what is the length of BC, in cm?



Answer **:** ____________________ cm

10. In a class, five students are on duty every day. Over a period of 30 school days, every two students will be on duty together on exactly one day. How many students are in the class?

Answer **:** ____________________ students

11. A committee is to be chosen from 4 girls and 5 boys and it must contain at least 2 girls. How many different committees can be formed?

Answer **:** ____________________ ways

12. Find the largest positive integer such that none of its digits is 0, the sum of its digits is 16 but the sum of the digits of the number twice as large is less than 20.

Answer **:** ____________________

Section B.

Answer the following 3 questions. Show your detailed solution on the space provided after each question. Each question is worth 20 points.

1. What is the number of ordered pairs (x, y) of positive integers such that $\frac{3}{x}+\frac{1}{y}=\frac{1}{2}$ and $\sqrt{xy}\geq 3\sqrt{6}$?

***Answer* :** ____________________

2. What is the minimum number of the 900 three-digit numbers we must draw at random such that there are always seven of them with the same digit-sum?

Answer **:** ____________________

3. Point M is the midpoint of the semicircle of diameter AC. Point N is the midpoint of the semicircle of diameter BC and P is midpoint of AB.
Prove that $\angle PMN = 45°$.