Invitational World Youth Mathematics Intercity

Individual Contest

Time limit: 120 minutes

Instructions:

- Do not turn to the first page until you are told to do so.
- Remember to write down your team name, your name and contestant number in the spaces indicated on the first page.
- The Individual Contest is composed of two sections with a total of 120 points.
- Section A consists of 12 questions in which blanks are to be filled in and only **ARABIC NUMERAL** answers are required. For problems involving more than one answer, points are given only when **ALL** answers are correct. Each question is worth 5 points. There is no penalty for a wrong answer.
- Section B consists of 3 problems of a computational nature, and the solutions should include detailed explanations. Each problem is worth 20 points, and partial credit may be awarded.
- You have a total of 120 minutes to complete the competition.
- No calculator, calculating device, electronic devices or protractor are allowed.
- Answers must be in pencil or in blue or black ball point pen.
- All papers shall be collected at the end of this test.

English Version

Team:______________ ***Name***:__________________ ***No.***: _____________ ***Score:***_________

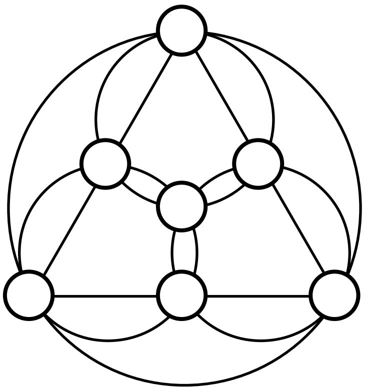
For Juries Use Only

No.	Section A												Section B			Total	Sign by Jury
	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3		
Score																	
Score																	

Section A.

In this section, there are 12 questions. Fill in the correct answer in the space provided at the end of each question. Each correct answer is worth 5 points.

1. Suppose we want to insert the integers 1 through 7 into the 7 little circles in the diagram below in such a way that the sum of the numbers written on each of the circumferences of the large and mid-sized circles equals a multiple of 6. What is the number to be inserted into the small circle in the center?



Answer : ______________

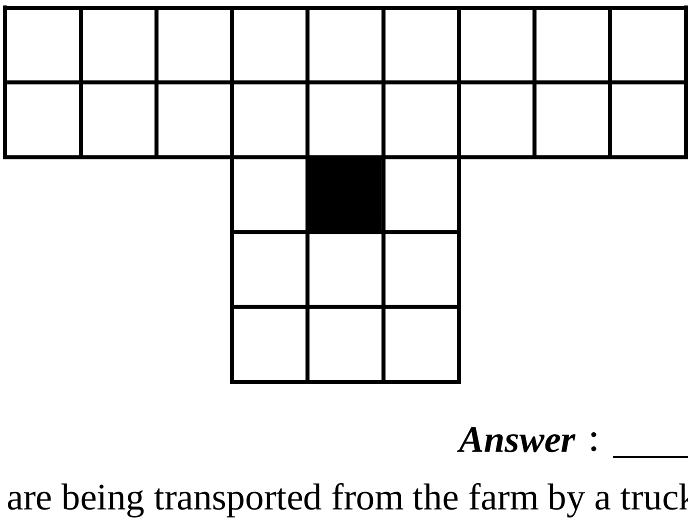
2. All digits of the positive integer *a* are distinct. The number *b* is obtained from *a* by rearranging its digits. If every digit of the difference $a-b$ is a 1, what is the largest value of this difference?

Answer : ____________

3. The largest of 21 distinct integers is 2015 and one of the other numbers is 101. The sum of any 11 of them is greater than the sum of the other 10. Find the middle number, that is, the one which is greater than 10 others and smaller than the remaining 10.

Answer : ____________

4. The diagram below consists of 26 squares around a black hole. How many different rectangles are there which consist of some of these 26 squares? The black hole must not be a part of any rectangle.



Answer : ____________rectangles

5. Some potatoes are being transported from the farm by a truck at a speed of

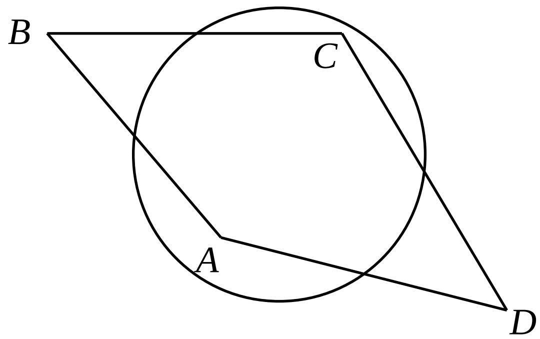
65 kph while the rest are being transported by an ox-cart at a speed at 5 kph. After a while, a horse-cart with speed 13 kph takes the potatoes off the truck, allowing the truck to go back and take the potatoes from the ox-cart. The transfer of potatoes from the ox-cart into the truck takes no time. The horse-cart and the truck arrive simultaneously in the city, which is 100 km from the farm. For how many hours have the potatoes been on the road?

Answer : ___________hours

6. How many positive integral solutions (x, y) are there for the equation $\frac{1}{x+1}+\frac{1}{y}+\frac{1}{(x+1)y}=\frac{1}{2015}$?

Answer : ___________

7. The midpoints of the sides *AB*, *BC*, *CD*, *DA* of a convex quadrilateral *ABCD* lie on the same circle. If $AB=10$ cm, $BC=11$ cm and $CD=12$ cm, determine, in cm, the length of the side *DA*.



Answer : ___________cm

8. The three two-digit numbers $\overline{ab}$, $\overline{cd}$ and $\overline{ad}$ are such that $(\overline{ab})^2+(\overline{cd})^2=(\overline{ad})^2$. Find the minimum value of the four-digit number $\overline{abcd}$.

Answer : ___________

9. Each 1×1 tile has a red side opposite a yellow side, and a blue side opposite a green side. An 8×8 chessboard is formed from 64 of these tiles, which may be turned around or turned over. When two tiles meet, the edges that come together must be of the same colour. How many different chessboards can be formed? Turning the chessboard around or turning the chessboard over is not allowed.

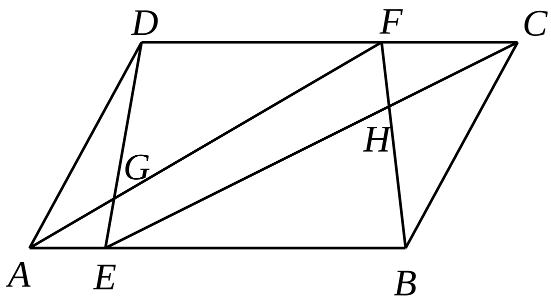
Answer : ___________chessboards

10. Find the sum of all the positive integers which can be expressed as $\sqrt{7p^n+9}$ for some positive integer n and some prime number p.

Answer : ___________

11. *ABCD* is a parallelogram. *E* is a point on the segment *AB* such that $\frac{AE}{EB}=\frac{1}{4}$. *F* is a point on the segment *DC*, *AF* and *DE* intersect at *G*, while *CE* and *BF*

intersect at H. If the area of $ABCD$ is 1 cm^2 and the area of triangle BHC is $\frac{1}{8}\text{cm}^2$, find the area, in cm^2, of triangle ADG.



Answer : ___________cm^2

12. Five different positive integers are such that if we take any two of them, possibly the same number twice, exactly nine different sums may be obtained. Find the largest positive integer which can divide the sum of any five such numbers.

Answer : ___________

Section B.
Answer the following 3 questions, and show your detailed solution in the space provided after each question. Each question is worth 20 points.

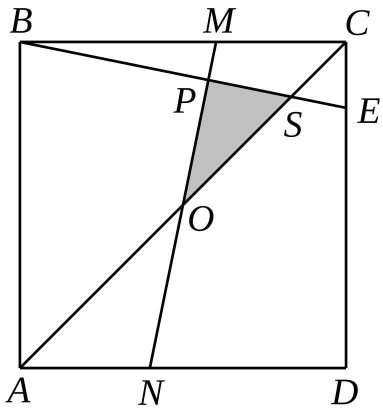
1. Find the number of the five-digit numbers that are perfect squares and have two identical digits in the end.

Answer : ___________

2. A number consists of three distinct digits chosen at random from1, 2, 3, 4, 5, 6, 7, 8 and 9 and then arranged in descending order. A second number is constructed in the same way except that the digit 9 may not be used. What is the probability that the first number is strictly greater than the second number?

Answer : ___________

3. E and N are points on the sides DC and DA of the square $ABCD$ such that $AN : ND : DE = 2 : 3 : 4$. The line through N perpendicular to BE cuts BE at P and BC at M. AC cuts MN at O and BE at point S. What fraction of the area of $ABCD$ is the area of triangle OPS?



Answer : ___________